Physics 195 / Applied Physics 195 — Assignment #10

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Due: **5pm SHARP (4:50pm + 10 min grace period)**, December 21st, 2015 @ MD Rm 131.

Do only Problems 1 ~ 3 for grading. Problem 4 is a study guide for Lecture #25.

Problem 1 (200 pt): Plasmonic excitation in gated 2D conductor

(NOTE) In this problem, $\vec{q}$ is for plasmonic wave vector, while $\vec{k}$ is for Bloch electron wave vector.

A 2D conductor is electrostatically electron-doped by a metallic gate at distance d. The equilibrium conduction electron density is n_0. Assume $T \ll T_F$ and also Fermi level well above the conduction band edge. The conduction band valley degeneracy is g_v, and the electron energy dispersion around the i-th valley centered at $\vec{k}_{0,i}$ ($i = 1, 2, ..., g_v$) is given by

$$\epsilon(\vec{k}) = A|\vec{k} - \vec{k}_{0,i}|^{\eta} \tag{1}$$

where the positive constants A and η are the electronic band properties of the conductor. The electric permittivity of the insulator sandwiched between the conductor and gate is ϵ_b, whose dispersion we will ignore. Consider plasmonic excitation in this 2D conductor. By consideirng both Coulomb and Pauli restoring forces (while not correcting the Pauli force for the high-frequency dynamical effect) and by assuming that the plasmonic wavelength is much longer than d, derive the plasmonic wave equation and show that the plasmonic dispersion is given by

$$\omega(\vec{q}) = \sqrt{\frac{g_v e^2 d}{\pi \epsilon_b} \times \frac{A\eta}{2\hbar^2}\left(\frac{2\pi n_0}{g_v}\right)^{\eta/2} + \frac{(A\eta)^2}{2\hbar^2}\left(\frac{2\pi n_0}{g_v}\right)^{\eta-1}} \times |\vec{q}|. \tag{2}$$

Show that the second term inside the square root in Eq. (2) is always equal to $v_F^2/2$ regardless of A and γ. For correcting this second term (associated with the Pauli force) to take into account the high-frequency effect, refer to Lectures # 22, 23, and 25 (this correction is not part of the present problem).

Problem 2 (100 pt): Charge screening in degenerate electron gas

An external point charge Q is inserted into a metal whose conduction electron density is n_0 and chemical potential is μ_0. The electrostatic dielectric constant due to bound electrons—the familiar dielectric constant that does not take into account the conduction electron effect—is κ. Calculate the overall steady state potential (screened potential) as a function of position $\vec{r}$, which is established after inserting this charge, by using the self-consistent field method as found in Lecture #24 (not the more formal one in Lecture #25). In this self-consistent field calculation, do include the effect of atomic polarization due to bound electrons. Calculate the charge screening length for copper, gold, and silver.

Problem 3 (100 pt): T-dependency of electrical resistivity due to electron-phonon scattering

(a) Recall from Assignment #4 that the average number of thermal phonons, n_{ph}, exhibits the following temperature dependency:

$$n_{ph} \propto \begin{cases} T^3 & (T \ll \Theta_D); \\ T & (T \gg \Theta_D). \end{cases} \tag{3}$$

In connection with this, show that the average energy per thermal phonon is given approximately by

$$\epsilon_{ph} \approx \begin{cases} k_B T & (T \ll \Theta_D); \\ k_B \Theta_D & (T \gg \Theta_D). \end{cases} \tag{4}$$

while omitting the proportionality constants on the order of 1.

(b) **Metal resistivity (due to electron-phonon scattering) at low temperatures**: Assume $T \ll \Theta_D$. Argue that the average thermal phonon momentum $p_{ph} = \hbar k_{ph}$ is far smaller than the electron Fermi momentum $p_F = \hbar k_F$. Therefore, one-time electron-phonon collision will cause only a very small angle scattering, which is too small to be directly associated with the Drude momentum relaxation time τ. These small-angle scatterings should accumulate through multiple collisions (say, r times) to cause one *effective* large angle (say, $90°$) electron scattering, which is directly associated with τ. Since the number of individual collisions (that give small angle scatterings) is proportional to n_{ph}, $\tau \propto r/n_{ph}$. Show that r is on the order of $\sim k_F^2/k_{ph}^2$, and is proportional to T^{-2}. Prove $\rho \propto T^5$ for metal.

(c) **Metal resistivity (due to electron-phonon scattering) at high temperatures**: Assume $T \gg \Theta_D$. Show that $p_{ph} \sim p_F$, and hence, $r \sim 1$ and $\rho \propto T$ for metal.

(d) **Semiconductor resistivity**: In metals, ρ increases with T with characteristic power laws as seen above. Briefly discuss how ρ changes with T in semiconductors.

Problem 4 (Study guide for Lecture #25): Collective electron response via Lindhardt longitudinal dielectric function

As derived in Lecture #25 using the self-consistent field method, Lindhardt's longitudinal dielectric function for a Fermi gas is given by

$$\frac{\epsilon(q,\omega)}{\epsilon_0} = 1 - \frac{e^2}{q^2\epsilon_0} \times \sum_{\vec{k},spin} \frac{f(\vec{k}+\vec{q}) - f(\vec{k})}{\epsilon(\vec{k}+\vec{q}) - \epsilon(\vec{k}) - \hbar\omega - i\hbar\delta}. \tag{5}$$

(a) From Eq. (5), derive the following dispersion relation for the longitudinal plasmonic excitation in a 3D Fermi gas

$$\omega(\vec{q}) \approx \omega_p \left[1 + \frac{3v_F^2}{10\omega_p^2} q^2 + \cdots \right], \tag{6}$$

and check consistency with Lecture #22.

(b) Using Eq. (5), show that a point charge Q placed in a metal will have a screened potential given by

$$\phi(r) = \frac{Q}{4\pi\epsilon_0} \frac{e^{-q_0 r}}{r} \tag{7}$$

with q_0, the inverse of the screening length, being given by

$$q_0^2 = \frac{e^2}{\epsilon_0} \times \frac{3n_0}{2\epsilon_F}. \tag{8}$$

Check consistency with the Thomas-Fermi screening calculated in Lecture #24.

(c) Using Eq. (5), show that the dispersion relation for the plasmonic excitation in an ungated 2D Fermi gas placed in free space can be approximated as

$$\omega^2(\vec{q}) \approx \frac{n_0 e^2}{2m_0\epsilon_0} q + \frac{3}{4} v_F^2 q^2 + \cdots. \tag{9}$$

Check consistency with Lecture #23.

(d) Show from Eq. (5) that the longitudinal plasmonic excitation *can* damp for $v_\parallel \leq v_F$ ($v_\parallel$: the phase velocity of the longitudinal plasmonic wave). Briefly describe the underlying physical mechanism for the damping.