Physics 195 / Applied Physics 195 — Assignment #11

Professor: Donhee Ham

Problem 1: Optical phonon polaritons in ionic crystal

NaCl crystal has a static dielectric constant of ~ 6. Calculate the frequency bandwidth within which optical phonon polariton propagation is forbidden and thus NaCl strongly reflects the incident electromagnetic radiation. Estimate $\epsilon(\infty)$. In solving this problem, you should estimate the force constant between an Na atom and a Cl atom from the static dielectric constant.

Problem 2: Electron paramagnetic resonance with Cr^{3+} in Ruby

A ruby sample—Al_2O_3 crystal with diluted Cr^{3+} ions (0.01%)—is placed in a static magnetic field $B_0 = 0.6$ T. This field is directed along the crystalline c axis. The sample temperature is 4.2 K.

(a) By taking into account the orbital quenching by the crystal field, estimate the Landé g factor for the incomplete 3d-shell of the Cr^{3+} ion. Argue that there are 4 Zeeman levels ($m_j = \pm 3/2, \pm 1/2$). Even in the absence of an external magnetic field, the $m_j = \pm 1/2$ states have a higher energy than the $m_j = \pm 3/2$ states, with the energy difference corresponding to a frequency of ~ 11 GHz. Considering this zero-field splitting, calculate all possible EPR frequencies at $B_0 = 0.6$ T.

From here on, we consider only the $m_j = -3/2$ and $m_j = -1/2$ states with $B_0 = 0.6$ T. Their energy difference is denoted as $\hbar\omega_0$, whose numerical value you calculated above. Assume spin-spin relaxation time of $T_2 \approx 1 \times 10^{-8}$ s and spin-lattice relaxation time of $T_1 \approx 3 \times 10^{-8}$ s. We apply an RF magnetic field with an amplitude of B_1 along the direction perpendicular to the c axis with the RF frequency ω tuned at ω_0.

(b) Assume $B_1 = 1$ G.

- Calculate the Rabi frequency ω_1. Calculate the stimulated transition rate W from the Fermi's golden rule. Check the satisfaction of $W = (\omega_1^2/2) \times T_2$, an important identity we derived in Lecture #28.
- Is the resonant transition in the coherent Rabi flopping regime or in the rate equation regime? Calculate the characteristic bandwidth of the resonant transition; that is, how much can the excitation frequency ω deviate from ω_0 while an appreciable resonant transition is maintained?
- If the irradiation of the RF magnetic field is sustained, how long will it take for the system to reach saturation? Estimate the saturated magnetization.

(c) Repeat Part (b) for $B_1 = 0.05$ T.

(d) Let W_b be the stimulated transition rate due to the background thermal noise (radiative + non-radiative) and δ_s be the spontaneous decay rate (radiative + non-radiative). Calculate W_b and δ_s. You should see that W_b is a far more dominant contributor (than δ_s) to the T_1 relaxation (this drastically contrasts the case of optical transitions). Explain why.

Problem 3: Magnetic order

(a) Compare the estimated strength of the dipolar and exchange interaction of two adjacent Fe atoms in Fe metal.

(b) Apply the Weiss's effective molecular field theory for ferromagnetic materials (Lecture #30) to an anti-ferromagnet and calculate the critical temperature (Néel temperature) above which the anti-ferromagnetic order vanishes.

Problem 4: Spin waves and thermal magnons

(a) By applying the ferromagnetic spin-wave formalism of Lecture #31, show that the dispersion relation of the anti-ferromagnetic spin waves is given by

$$\omega = 2\frac{SJ}{\hbar^2}|\sin(ka)|, \tag{1}$$

where a is the distance between the two neighboring spins with opposite directions.

(b) A static magnetic field B_0 is applied perpendicularly to a thin ferromagnetic film (inter-spin distance: a), inducing a magnetization of M_0. Show that the spin wave dispersion relation in the long wavelength regime is given by

$$\omega \approx \frac{SJa^2}{\hbar^2}k^2 + |\gamma_e|(B_0 - \mu_0 M_0). \tag{2}$$

(c) Consider thermal excitation of spin waves (thermal magnons) in ferromagnetic materials. Assume low enough temperature for which the ferromagnetic spin wave dispersion of Lecture #31 can be approximated into the form $\omega \sim k^2$. Calculate the average number of thermal magnons and show that it is proportional to $T^{3/2}$. Furthermore, show that at these low temperatures, the increasing excitation of thermal magnons with temperature reduces the spontaneous magnetization M according to

$$M(T) \approx M(0)\left[1 - \alpha T^{3/2}\right] \tag{3}$$

where α is a certain constant. Calculate α. Finally, calculate the total energy of thermal magnons at T and show that the associated heat capacity is proportional to $T^{3/2}$.

Problem 5: Ferromagnetic and ferrimagnetic microwave devices

(a) An yttrium-iron-garnet (YIG) sphere has a magnetization of $\mu_0 M$ 0.18 T in a static magnetic field of $B_0 = 0.12$ T. Calculate the ferromagnetic resonance frequency as well as the magnetic field inside the YIG sphere.

(b) A static magnetic field B_0 of 0.2 T is applied along the y direction through a ferrite material, which then exhibits a saturation magnetization of $\mu_0 M = 0.12$ T. Its dielectric constant is 5. Two linearly polarized microwave plane waves (angular frequency: ω) propagate along the z direction. One is polarized along the y direction; the other along the x direction. Calculate the phase difference between the two microwaves as a function of the propagation distance.