Applied Physics 195 / Physics 195 — Assignment #3

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Date: October 2nd, 2015
Due: **5pm SHARP (4:50pm + 10 min grace period)**, October 9th, 2015; slide your work under through the door at Maxwell-Dworkin Room 131.

Problem 1 (50 pt; NO COLLABORATION): Mode counting for lattice waves

Consider a 2D rectangular crystal—2D both structurally and dynamically—with sides L_x and L_y. Its square unit cell with side a contains p atoms.

(a) Calculate the area of the first Brillouin zone and the number of allowed $\vec{k}$ vectors within the first Brillouin zone.

(b) For a given $\vec{k}$ vector, how many lattice wave modes exist? In other words, how many branches of dispersion relation are there? Out of these, how many are acoustic branches? What is the total number of possible lattice wave modes?

(c) Let $\omega_s(\vec{k})$ describe the s-th branch lattice wave dispersion relation ($s = 1, 2, ..., s_{max}$ with s_{max} being the number of dispersion relation branches). We denote the corresponding phonon energy density of states as $D_s(\omega)$. Evaluate $\int d^2\vec{k}/(2\pi)^2$ (the integration is over the first Brillouin zone) and $\sum_{s=1}^{s_{max}} \int D_s(\omega)d\omega$ (the integration is over the frequency range corresponding to the first Brillouin zone).

Problem 2 (50 pt; NO COLLABORATION): Debye model for 2D crystal

Consider a 2D crystal—2D both structurally and dynamically—with one atom per unit cell. Assume a crystal area of A and a total of N atoms in the crystal. We apply the Debye model to this crystal. For all possible branches of lattice wave dispersion relations, use the identical $\omega(\vec{k}) = v_0 k$, with v_0 serving as the speed of sound. Calculate the phonon energy density of states for each branch of dispersion relation. Calculate the specific heat due to the lattice waves at a general temperature T. Verify that for $T \gg \Theta_D$ (Θ_D: Debye temperature), the specific heat converges to that described by the 2D version of Dulong-Petit law. Calculate the specific heat for $T \ll \Theta_D$, and show that it is proportional to T^2.

Problem 3 (50 pt; NO COLLABORATION): Debye model for 1D crystal

Consider now a 1D crystal—1D both structurally and dynamically—with one atom per unit cell. Assume a crystal length of L and a total of N atoms in the crystal. We once again apply the Debye model to this 1D crystal. Let the lattice wave dispersion relation be approximated as $\omega(\vec{k}) = v_0 k$, with v_0 being the sound speed. Calculate the phonon energy density of states. Calculate the specific heat due to the lattice waves at a general temperature T. Verify that for $T \gg \Theta_D$, the specific heat agrees with the 1D version of Dulong-Petit law. Show that the specific heat for $T \ll \Theta_D$ is proportional to T.

Problem 4 (50 pt; NO COLLABORATION): Spectral range of acoustical lattice waves

From the Debye temperature table on page 12 of Lecture Note # 5, estimate the spectral ranges of the acoustical lattice wave modes of the solids listed in the table.