Applied Physics 216 — Assignment #7

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Due: **10:20am + 10 min grace period**, April 7th, 2017; slide your work under through the door at Maxwell-Dworkin Room 131.

Problem 1 (250 pt): Fabry-Perot Etalon

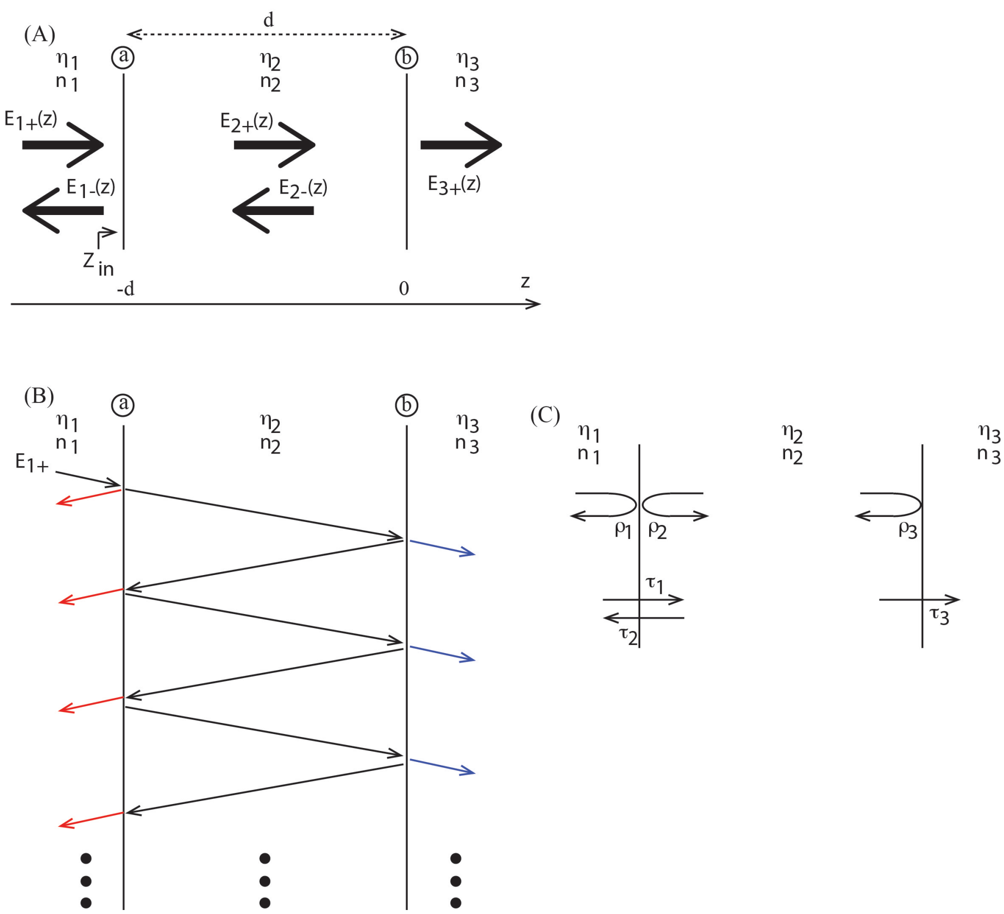

Figure 1: Fabry-Perot Etalon: (A) Overall forward and reverse light waves. (B) Infinite number of local reflections and transmissions. While all waves travel along $\pm\hat{z}$ axis, we slant light lines not to crowd the space. (C) Local reflection and transmission coefficients.

A dielectric layer of thickness d (intrinsic impedance: η_2; refractive index: n_2) is sandwiched between a dielectric of intrinsic impedance η_1 and refractive index n_1 to the left and a dielectric of intrinsic impedance η_3 and refractive index n_3 to the right [Fig. 1]. An x-polarized light wave is normally impinged from the first dielectric along the $+\hat{z}$ axis. Figure 1(A) shows the resulting, *overall* forward and reverse light waves in each layer in steady state. The fields of these overall light waves are superpositions of infinite numbers of fields of locally reflected and transmitted waves [Fig. 1(B)] where the local reflection and transmission coefficients are shown in Fig. 1(C). Concretely, when the incident light first hits boundary ⓐ, it is partly reflected with coefficient ρ_1, and the rest is transmitted with coefficient τ_1 onto the 2nd dielectric.[1] The transmitted wave then travels distance d in the 2nd dielectric toward boundary ⓑ, where it is partially transmitted with coefficient τ_3 and partially reflected with coefficient ρ_3. This reflected wave then travels another distance d back to boundary ⓐ. Part of this wave is transmitted to the first dielectric with coefficient τ_2, and the

[1]Here ρ_1 and τ_1 are determined completely by n_1 and n_2 without involving n_3 (or equivalently by η_1 and η_2 without involving η_3) because the light has not yet traveled to the 3rd dielectric — in this sense, these coefficients are "local".

rest is reflected with coefficient ρ_2. These local reflection and transmission processes continue, generating an infinite number of partial waves. The total electric field E_{1-} of the overall reflected light in the first dielectric [Fig. 1(A)] is then the superposition of the electric fields of the infinite number of partial light waves shown as red lines in Fig. 1(B). Similarly the total electric field E_{3+} of the overall transmitted light in the third dielectric [Fig. 1(A)] is the superposition of the electric fields of the infinite number of partial light waves shown as blue lines in Fig. 1(B).

(a) By adding up the electric fields of all of the partial waves in the third dielectric shown as blue lines in Fig. 1(B), show that the ratio of the *overall* intensity of the light transmitted into the third dielectric to the intensity of the incident light in the first dielectric is given by

$$\frac{I_t}{I_i} = \frac{(1-\rho_2^2)(1-\rho_3^2)}{(1-\rho_2\rho_3)^2 + 4\rho_2\rho_3\sin^2(k_2 d)}, \tag{1}$$

where k_2 is the wavenumber in the second dielectric. Similarly, by adding up the electric fields of all of the partial waves in the first dielectric shown as red lines in Fig. 1(B), show that the ratio of the *overall* intensity of the light reflected into the first dielectric to the intensity of the incident light in the first dielectric is given by

$$\frac{I_r}{I_i} = \frac{(\rho_2-\rho_3)^2 + 4\rho_2\rho_3\sin^2(k_2 d)}{(1-\rho_2\rho_3)^2 + 4\rho_2\rho_3\sin^2(k_2 d)}. \tag{2}$$

Check $(I_t + I_r)/I_i = 1$. Also check that Eqs. (1) and (2) reduce to the results of Lecture #17, if $n_1 = n_3$.

(b) We can obtain Eqs. (1) and (2) without resorting to the partial waves and their summation, but by strictly considering the *overall* waves. Let's derive Eq. (2) first. As a first step, show that the impedance looking into second dielectric at boundary ⓐ is given by

$$Z_{in} = \left.\frac{E_{2+}(z) + E_{2-}(z)}{H_{2+}(z) + H_{2-}(z)}\right|_{z=-d} = \eta_2 \frac{e^{ik_2 d} + \rho_3 e^{-ik_2 d}}{e^{ik_2 d} - \rho_3 e^{-ik_2 d}} = \eta_2 \frac{\eta_3\cos(k_2 d) + i\eta_2\sin(k_2 d)}{\eta_2\cos(k_2 d) + i\eta_3\sin(k_2 d)}. \tag{3}$$

Subsequently, using Z_{in} above, calculate the *overall* reflection coefficient (from the first dielectric back to the first dielectric) at boundary ⓐ, $\rho_{overall} = E_{1-}(z)/E_{1+}(z)|_{z=-d}$.

(c) Building up from the result of part (b) and continuing to consider only the *overall* waves, derive Eq. (1) (do *not* use $(I_t + I_r)/I_i = 1$, *i.e.*, we won't assume this *a priori*).

(d) Before we look at the Fabry-Perot etalon (η_1, $\eta_3 \ll \eta_2$ or equivalently n_1, $n_3 \gg n_2$), let's explore some important implications of the results above.

- Show that if $n_2 = \sqrt{n_1 n_3}$ and d is a quarter wavelength, $I_r = 0$ and $Z_{in} = \eta_1$ (impedance matching). So in this case, the second layer serves as an anti-reflection coating, where it creates a perfect impedance matching to prevent reflection.
- Show that if d is a half wavelength, $Z_{in} = \eta_3$. That is, the second layer becomes "transparent", and at boundary ⓐ, the intrinsic impedance of the third dielectric is presented as an input impedance, as if the second layer did not exist. In this case, the second layer serves as a electromagnetically transparent coating that can mechanically or chemically protect the third dielectric layer.
- Show that if the third dielectric is an effective short circuit ($\eta_3 = 0$) and d is a quarter wavelength, boundary ⓐ presents an open circuit ($Z_{in} = \infty$) to the first dielectric.
- Show that if the third dielectric is an effective open circuit ($\eta_3 = \infty$) and d is a quarter wavelength, boundary ⓐ presents a short circuit ($Z_{in} = 0$) to the first dielectric.

(e) Now we consider the Fabry-Perot etalon (η_1, $\eta_3 \ll \eta_2$ or equivalently n_1, $n_3 \gg n_2$). For simplicity, assume $n_1 = n_3$ and define $R = \rho_2\rho_3$. Show that I_t/I_i can be re-expressed as

$$\frac{I_t}{I_i} = \frac{1}{1 + (2F/\pi)^2 \sin^2(k_2 d)} \tag{4}$$

where F, the finesse, is defined as

$$F = \frac{\pi\sqrt{R}}{1-R}. \tag{5}$$

Sketch F versus R ($0 \leqslant R \leqslant 1$) to check that a larger R (less reflective loss) corresponds to a larger finesse. Evaluate the finesse for $R = 0.9$, 0.95, and 0.99, and for each of these finesse values, plot I_t/I_i as a function of frequency.

(f) While we have not assumed any loss in the dielectric media, there is a reflective loss (from the point of view of Fabry-Perot etalon/resonator), if R is smaller than 1. Assuming $d = 15$ cm and $n_2 \approx 1$, calculate the quality factor Q (and compare it to the finesse F), the photon lifetime τ_p, and the total attenuation coefficient α_{tot} at standing wave resonance modes of the etalon near a wavelength 633 nm for $R = 0.9$, 0.95, and 0.99. What is the value of the free spectral range?

(g) Repeat part (f), now assuming that the second dielectric introduces a loss with attenuation coefficient of $\alpha = 1$ m^{-1}.